REZUMAT

ANA-LOREDANA AGORE

Am obtinut titlul de doctor la Vrije Universiteit Brussel, Belgia in Octombrie 2012 cu cea mai inalta distinctie din sistemul belgian. In prezent detin un grant postdoctoral oferit de Flemish Science Foundation (FWO-Vlandeeren) la aceeasi universitate precum si un post de lector la Academia de Studii Economice, Bucuresti. Pe toata perioada studiilor doctorale (2010-2012) precum si in perioada 2012-2014 am beneficiat de un grant Aspirant oferit de Flemish Science Foundation (FWO-Vlandeeren). De asemenea, am facut parte din echipa de cercetare a altor trei granturi finantate de CNCS-UEFISCDI.

Articolele stiintifice pe care le-am scris sunt publicate in reviste cotate ISI din strainatate, din care mentionez: Ann. Inst. Fourier, J. Noncommutative Geom., Monatsh. für Mathematik, Proc. Amer. Math. Soc., J. Math. Phys., J. Algebra, Linear Algebra Appl., Algebr. Represent. Theory., Appl. Cat. Structures, Linear Multilinear Algebra, Glasgow Math. J., etc. Rezultatele obtinute au fost prezentate la numeroase conferinte internationale in Franta, Anglia, Italia, Spania, Belgia, Cehia, Polonia, Turcia etc. In Decembrie 2013, am obtinut premiul Gheorghe Ţiţeica al Academiei Romane pentru contributii la studiul algebrelor Hopf. Am fost cercetator invitat la Max Planck Institut für Mathematik (Bonn Germany), Institut des Hautes Études Scientifiques (Paris, France), University of Copenhagen (Copenhagen, Denmark) etc.

Aceasta teza contine o parte a rezultatelor obtinute de autor privind *problema extinderii structurilor* precum si alte probleme inrudite cum ar fi: problema de factorizare, problema clasificarii complementilor si problema extinderilor. Problema mai sus mentionata a fost introdusa in [15] in contextul foarte general al teoriei categoriilor:

Problema extinderi structurilor (ES): *Fie $F : \mathcal{C} \to \mathcal{D}$ un functor uituc si consideram doua obiecte $C \in \mathcal{C}$, $D \in \mathcal{D}$ astfel incat $F(C)$ este un subobject al lui D in $\mathcal{D}$. Descrieti si clasificati toate structurile matematice (S) ce se pot defini pe D astfel incat D sa devina un obiect in $\mathcal{C}$ si C un subobject al lui D in categoria $\mathcal{C}$ (clasificarea este pana la un izomorfism care stabilizeaza C si un anumit tip de factor D/C fixat).*

Primul capitol trateaza aceasta problema in contextul algebrelor Leibniz / Lie. Fie $\mathfrak{g}$ o algebra Leibniz si E un spatiu vectorial ce contine $\mathfrak{g}$ ca si subspatiu. Toate structurile de algebra Leibniz ce se pot defini pe E astfel incat $\mathfrak{g}$ sa devina subalgebra Leibniz sunt descrise explicit si clasificate prin doua obiecte ne-abeliene de tip coomologic: $\mathcal{HL}^2_{\mathfrak{g}}(V, \mathfrak{g})$ furnizeaza clasificarea pana la un izomorfism ce stabilizeaza $\mathfrak{g}$ iar $\mathcal{HL}^2(V, \mathfrak{g})$ clasifica toate aceste structuri din punctul de vedere al problemei clasice a extinderii - aici V este un complement al lui $\mathfrak{g}$ in E. Un produs general, numit produs unified, este introdus ca unealta pentru abordarea folosita. Produsul crossed (resp. bicrossed) a doua algebre Leibniz sunt introduse ca si cazuri speciale ale produsului unified: primul dintre aceastea este responsabil pentru problema clasica a extinderii in timp ce produsul bicrossed este responsabil pentru problema factorizarii. Partea ce se ocupa de algebre Leibniz din acest capitol se incheie cu o scurta analiza a teoremei Itô, un rezultat binecunoscut in teoria grupurilor. Se arata ca rezultatul ramane valid si in contextul algebrelor Leibniz: daca $\mathfrak{g}$ este o algebra Leibniz astfel incat $\mathfrak{g} = A + B$, pentru doua subalgebre abeliene A si B, atunci $\mathfrak{g}$ is metabeliana, i.e. $[\,[\mathfrak{g},\, \mathfrak{g}],\, [\mathfrak{g},\, \mathfrak{g}]\,] = 0$.

Rezultatele de mai sus privind problema extinderii structurilor pentru algebre Leibniz sunt specializate pentru algebre Lie. Rezultatele corespunzatoare sunt enuntate fara demonstratie si numeroase exemple in care se calculeaza explicit cele doua obiecte de clasificare $\mathcal{H}^2_{\mathfrak{g}}(V, \mathfrak{g})$ si $\mathcal{H}^2(V, \mathfrak{g})$ sunt prezentate in detaliu in cazul extinderilor de tip steag. Rezultatele prezentate in acest capitol sunt continute in articolele [17], [19] si respectiv [22].

Cel de-al doilea capitol are ca tema un caz special al problemei ES, si anume problema factorizarii precum si reciproca acesteia, problema clasificarii complementilor. Vom enunta ambele probleme in cel mai general context al teoriei categoriilor. Vom spune ca un obiect $E \in \mathcal{C}$ *factorizeaza* prin A si H daca E poate fi scris ca un "produs" al lui A si H, unde A si H sunt subobiecte ale lui E avand intersectie minimala. Aici, semnificatia cuvantului "produs" depinde de natura categoriei in care lucram. Un subobiect H al lui E se va numi *complement* al lui A in E (sau A-*complement* al lui E) daca E poate fi scris ca un "produs" al lui A si H astfel incat A si H au "intersectie minimala" in E. In acest context, daca in problema ES adaugam ipoteza suplimentara ca "complementul lui A in E safe izomorf cu un obiect dat H" obtinem problema de factorizare care poate fi enuntata explicit astfel:

Problema factorizarii. *Fie A si H doua obiecte fixate in $\mathcal{C}$. Descrieti si classificati pana la izomorfism toate obiectele E din $\mathcal{C}$ care factorizeaza prin A si H.*

Daca notam prin $[E : A]^f$ cardinalul claselor de izomorfism ale tuturor A-complementilor lui E si il numim *index de factorizare* al lui A in E, urmatoarea problema naturala apare:

Problema clasificarii complementilor (PCC): *Fie $A \subset E$ un subobiect fixat al lui E in $\mathcal{C}$. Daca un A-complement al lui E exista, descrieti explicit, clasificati toti A-complementii lui E si calculati indexul de factorizare $[E : A]^f$.*

Prima parte a acestui capitol trateaza problema factorizarii in contextul algebrelor Lie. Mai precis, pentru o algebra Lie perfecta $\mathfrak{h}$ clasificam toate algebrele Lie ce contin $\mathfrak{h}$ ca subalgebra de codimensiune 1. Grupurile de automorfisme ale acestor algebre Lie sunt determinate explicit ca subgrupuri in produsul semidirect $\mathfrak{h} \ltimes \big(k^* \times \mathrm{Aut}_{\mathrm{Lie}}(\mathfrak{h})\big)$. In cazul non-perfect clasificarea acestor algebre Lie este o misiune dificila. Fie $\mathfrak{l}(2n+1, k)$ algebra Lie cu bracketul $[E_i,\, G] = E_i$, $[G,\, F_i] = F_i$, pentru orice $i = 1, \cdots, n$. Sunt descrise explicit toate algebrele Lie ce contin $\mathfrak{l}(2n+1, k)$ ca subalgebra de codimensiune 1 prin calculul tuturor produselor bicrossed $k \bowtie \mathfrak{l}(2n+1, k)$. Acestea sunt parametrizate de o multime de matrice $\mathrm{M}_n(k)^4 \times k^{2n+2}$ ce sunt descrise explicit. Numeroase deformari de tip pereche potrivita a algebrei Lie $\mathfrak{l}(2n+1, k)$ sunt descrise cu scopul de a calcula indexul de factorizare al anumitor extinderi de tipul $k \subset k \bowtie \mathfrak{l}(2n+1, k)$. De asemenea, este prezentat un exemplu de extindere avand indexul de factorizare infinit.

Exact ca in cazul algebrelor Lie [122, Theorem 4.1], [121, Theorem 3.9] vom introduce conceptul de pereche potrivita de algebre Leibniz si ii vom asocia produsul bicrossed care va fi responsabil pentru problema de factorizare. Totusi, in acest caz definitia conceptului de pereche portivita este mult mai elaborata si dificila decat cea pentru algebre Lie. Descrierea si clasificarea tuturor complementilor unei extinderi date $\mathfrak{g} \subseteq \mathfrak{E}$ de algebre Leibniz este prezentata ca o reciproca a problemei de factorizare. Acestia sunt clasificati de un alt obiect coomologic notat cu $\mathcal{HA}^2(\mathfrak{h}, \mathfrak{g} \,|\, (\triangleright, \triangleleft, \leftharpoonup, \rightharpoonup))$, unde $(\triangleright, \triangleleft, \leftharpoonup, \rightharpoonup)$ este perechea potrivita canonica asociata unui complement fixat $\mathfrak{h}$. Numeroase exemple sunt prezentate in detaliu.

In final, pentru a evidentia strategia folosita pentru a aborda problema clasificarii complementilor in contextul altor structuri algebrice, am inclus cateva rezultate privind algebrele asociative. Mai precis, primul capitol se termina cu un scurt sumar al problemei clasificarii complementilor in acest context. Materialul prezentat in acest capitol se bazeaza pe articole [20], [17] si respectiv [9].

Ce de-al treilea capitol trateaza un caz special al problemei extinderii structurilor in cazul algebrelor Hopf, si anume asa-numitele extinderi splitate de tip coalgebra. Fie A si H doua algebre Hopf date. O extindere splitata de tip coalgebra a lui A prin H este o pereche (E, π), unde E este o algebra Hopf ce se potriveste intr-un sir $A \hookrightarrow E \stackrel{\pi}{\to} H$ astfel incat morfismul de algebre Hopf $\pi : E \to H$ spliteaza in categoria de coalgebre si $A \simeq E^{\mathrm{co}(H)}$. Se arata ca orice extindere splitata de tip coalgebra a lui A prin H este echivalenta cu o extindere de tip produs crossed $(A\#H, \pi_H)$. In concluzie, clasificarea tuturor extinderilor splitate de tip coalgebra a lui A prin H se reduce la clasificarea produselor crossed $A\#_f^{\triangleright} H$ asociate tututror sistemelor crossed de algebre Hopf $(A, H, \triangleright, f)$. Produsul crossed este desigur un caz special de produs unified pentru algebre Hopf introdus in [15] ca raspuns la varianta restrictiva a problemei extinderii structurilor pentru algebre Hopf.

Pentru doua algebre Hopf A si H vom clasifica toate produsele crossed $A\#\, H$ calculand explicit doua obiecte de clasificare: 'grupul' coomologic $\mathcal{H}^2(H, A)$ si $\textsc{Crp}(H, A) :=$ multimea tipurilor de izomorfisme a tuturor prosuselor crossed $A\#\, H$. Toate produsele crossed de tipul $A\#H_4 := A_{(a\,|\,g,\,x)}$ sunt descrise prin generatori si relatii si clasificate: acestea sunt parametrizate de multimea $\mathcal{ZP}(A)$ elementelor centrale primitive ale lui A. Vom calcula $\mathcal{H}^2(H_4, A)$ si $\textsc{Crp}(H_4, A)$ pentru o clasa larga de algebre Hopf A. Numeroase exemple sunt descrise in detaliu: in particular, peste un corp de caracteristica $p \geq 3$ vom construi un exemplu de o familie infinita de algebre Hopf neizomorfe de dimensiune $4p$. Pentru grupul ciclic C_n, toate produsele crossed de forma $H_4\#k[C_n]$ sunt descrise si clasificate calculand $\mathcal{H}^2(k[C_n], H_4)$ si $\textsc{Crp}(k[C_n], H_4)$. Acestea sunt algebre Hopf $4n$-dimensionale $H_{4n,\,\lambda,\,t}$, asociate tuturor perechilor $(\lambda,\, t)$ constand intr-o functie unitara arbitrara $t : C_n \to C_2$ si o radacina λ de ordin n a lui ± 1. Grupul automorfismelor acestor algebre Hopf sunt de asemenea descrise. Acest capitol este inspirat din articolele [10] si respectiv [12] elaborate in colaborare cu G. Bontea si G. Militaru.

In cel de-al patrulea capitol sunt studiate algebrele Jacobi/Poisson, acestea fiind echivalentul algebric al varietatilor Jacobi/Poisson. Sunt introduse reprezentarile unei algebre Jacobi A si sunt definite algebrele Jacobi Frobenius. Este demonstrata o teorema de caracterizare a algebrelor Jacobi Frobenius ce foloseste notiunea de integrala pentru algebrele Jacobi. Pentru un spatiu vectorial V este introdus un obiect coomologic neabelian $\mathcal{JH}^2\,(V,\,A)$ care clasifica toate structurile de algebra Jacobi ce contin A ca si subagebra Jacobi de codimensiune egala cu $\dim(V)$. Reprezentarile algebrei Jacobi A sunt folosite pentru a obtine o descompunere a obiectului de clasificare $\mathcal{JH}^2\,(V,\,A)$ ca un coprodus peste toate structurile de A-modul Jacobi pe V. Produsul bicrossed $P \bowtie Q$ intre doua algebre Poisson introdus recent de Ni si Bai [42] este obtinut ca si caz special al constructiei noastre. Este introdus un nou tip de deformare a unei algebre Poisson Q si un obiect coomologic $\mathcal{HA}^2\big(P,\,Q \mid (\triangleleft,\,\triangleright,\,\leftharpoonup,\,\rightharpoonup)\big)$ este construit explicit ca o multime de clasificare pentru problema de descent a extinderilor de algebre Poisson. Numeroase exemple si aplicatii sunt deasemenea incluse. Acest capitol se bazeaza pe lucrarea [24] elaborata in colaborare cu G. Militaru.

Ultimul capitol descrie pe scurt directiile viitoare de cercetare ale autoarei.

References

[1] Abrams, L. - The quantum Euler class and the quantum cohomology of the Grassmannians, *Is. J. Math.*, **117**(2000), 335-352.
[2] Adem, A., R. J. Milgram, R. J. - Cohomology of finite groups, Berlin: Springer, 2nd Edition, 2004.
[3] Agaoka, Y. - An algorithm to determine the isomorphism classes of 4-dimensional complex Lie algebras, *Linear Algebra Appl.* **345** (2002), 85-118.
[4] Agore, A.L. - Categorical constructions, braidings on monoidal categories and bicrossed products of Hopf algebras, VUB Press, 199 + xx pages, ISBN 978-90-5718-212-9; PhD Thesis, Vrije Universiteit Brussel, 2012.
[5] Agore, A.L. - Limits of coalgebras, bialgebras and Hopf algebras, *Proc. Amer. Math. Soc.*, **139** (2011), 855–863
[6] Agore, A.L. - Categorical constructions for Hopf algebras, *Comm. Algebra.*, **39** (2011), 1476–1481
[7] Agore, A.L. - Crossed product of Hopf algebras, *Comm. Algebra*, **40** (2013), 2519 – 2542.
[8] Agore, A.L. - Free Poisson Hopf algebras generated by coalgebras, *J. Math. Phys.* **10** (2014), 083502.
[9] Agore, A.L. - Classifying complements for associative algebras, *Linear Algebra Appl.* **446** (2014), 345–355.
[10] Agore, A.L., Bontea, C.G., Militaru, G. – Classifying coalgebra split extensions of Hopf algebras, *J. Algebra Appl.* **12** (2013), no. 5, 1250227, 24 pp.
[11] Agore, A.L., Bontea, C.G., and Militaru, G. - Classifying bicrossed products of Hopf algebras, *Algebr. Represent. Theory*, **17** (2014), 227–264.
[12] Agore, A.L., Bontea, C.G., Militaru, G. – The classification of all crossed products $H_4\#k[C_n]$, *Symmetry Integrability Geom. Methods Appl.* **10** (2014), 049, 12 pages.
[13] Agore, A.L., Caenepeel, S., Militaru, G. - Braidings on the category of bimodules, Azumaya algebras and epimorphisms of rings, *Appl. Cat. Structures* **22** (2014), 29–42.
[14] Agore, A.L., Chirvasitu, A., Ion, B. and Militaru, G. - Bicrossed products for finite groups, *Algebr. Represent. Theory* **12** (2009), 481–488.
[15] Agore, A.L. and Militaru, G. - Extending structures II: the quantum version, *J. Algebra* **336** (2011), 321–341.
[16] Agore, A.L. and Militaru, G. - Classifying complements for Hopf algebras and Lie algebras, *J. Algebra*, **391** (2013), 193–208.
[17] Agore, A.L., Militaru, G. - Unified products for Leibniz algebras. Applications, *Linear Algebra Appl.* **439** (2013), 2609–2633.
[18] Agore, A.L. and Militaru, G. - Extending structures I: the level of groups, *Algebr. Represent. Theory* **17** (2014), 831–848.

[19] Agore, A.L. and Militaru, G. - Extending structures for Lie algebras, *Monatsh. für Mathematik*, **174** (2014), 169–193.

[20] Agore, A.L. and Militaru, G. - Bicrossed products, matched pair deformations and the factorization index for Lie algebras, *Symmetry Integrability Geom. Methods Appl.* **10** (2014), 065, 16 pages.

[21] Agore, A.L. and Militaru, G. - The global extension problem, crossed products and co-flag non-commutative Poisson algebras, *J. Algebra* **426** (2015), 1–31.

[22] Agore, A.L. and Militaru, G. - Itô's theorem and metabelian Leibniz algebras, 12 pages, to appear in *Linear Multilinear Algebra*, DOI:10.1080/03081087.2014.992771.

[23] Agore, A.L. and Militaru, G. - Classifying complements for groups. Applications, to appear in *Ann. Inst. Fourier.*

[24] Agore, A.L. and Militaru, G. - Jacobi and Poisson algebras, to appear in *J. Noncommutative Geom.*

[25] Agore, A.L., Militaru, G. - The extending structures problem for algebras, arXiv:1305.6022.

[26] Aguiar, M., Andruskiewitsch, N. - Representations of matched pairs of groupoids and applications to weak Hopf algebras. Algebraic structures and their representations, *Contemp. Math.* **376** (2005), 127173, Amer. Math. Soc., Providence.

[27] Albeverio, S., Omirov, B.A and Rakhimov, I.S. - Varieties of nilpotent complex Leibniz algebras of dimension less than five, *Comm. Algebra* **33** (2005) 1575-1585.

[28] Albeverio, S., Ayupov, Sh.A., Omirov, B.A. and Khudoyberdiyev, A.Kh. - n-Dimensional filiform Leibniz algebras of length $(n-1)$ and their derivations, *J. Algebra*, **319** (2008) 2471-2488.

[29] Alekseevsky, D., Michor, P. W. and Ruppert, W. - Extensions of Lie algebras. Unpublished. ESI Preprint 881. arXiv:math.DG/0005042.

[30] Alekseevsky, D., Michor, P. W. and Ruppert, W. - Extensions of super Lie algebras, *J. Lie Theory* **15** (2005), 125–134.

[31] Alperin, J.L. and Bell, R.R. - Groups and representations, Springer-Verlag, New York, 1995.

[32] Amberg, B., Franciosi, S. and de Giovanni, F. - Products of groups, Oxford University Press (1992).

[33] Andrada, A. and Salamon, S. - Complex product structures on Lie algebras, *Forum Mathematicum.*, **17** (2005), 261-295.

[34] Andrada, A., Barberis, M.L., Dotti, I.G. and Ovando, G.P. - Product structures on four dimensional solvable Lie algebras, *Homology, Homotopy and Applications*, **7**(2005), 9–37.

[35] Andruskiewitsch, N. - Notes on Extensions of Hopf algebras, *Can. J. Math.* **48** (1996), 3–42.

[36] Andruskiewitsch, N., Devoto, J. - Extensions of Hopf algebras, *Algebra i Analiz* **7** (1995), 22–61.

[37] Andruskiewitsch, N., Natale, S. - Examples of self-dual Hopf algebras, *J. Math. Sci. Univ. Tokyo*, **6**(1999), 181-215.

[38] Andruskiewitsch, N., Schneider, H.J. - Lifting of quantum linear spaces and pointed Hopf algebras of order p^3, *J. Algebra*, **209**(1998), 658–691.

[39] Ardizzoni, A., Beattie, M. and Menini, C. - Cocycle deformations for Hopf algebras with a coalgebra projection, *J. Algebra*, **324** (2010), 673–705.

[40] Ardizzoni, A., Menini, C. and Stefan, D. - A monoidal approach to splitting morphisms of bialgebras, *Trans. AMS*, **359** (2007), 991–1044

[41] Bahturin, Y. and Kegel, O. H. - Universal sums of abelian subalgebras, *Comm. Algebra*, **23** (1995), 2975 – 2990.

[42] Bai, C. - Double constructions of Frobenius algebras, Connes cocycles and their duality, *J. Noncommu. Geom.* **4** (2010), 475-530.

[43] Barnes, D. - On Levis Theorem for Leibniz algebras, *Bull. Australian Math. Soc.*, **86** (2012), 184-185.

[44] Benayadi, S. - Structure of perfect Lie algebras without center and outer derivation, *Annales de la faculte de Science de Toulouse*, **5**(1996), 203–231.

[45] Bloh, A.M. - On a generalization of the concept of Lie algebra, *Dokl. Akad. Nauk SSSR*, **165**(1965), 471-473.

[46] Böhm, G., Nill, F. and Szlachányi, K. - Weak Hopf algebras. I. Integral theory and C*-structure, *J. Algebra* **221** (1999), 385–438.

[47] Bourbaki, N. - Lie groups and Lie algebres, Chap. 1-3, Springer, Paris, 1989.

[48] Brzezinski, T., Hajac, P.M. - Coalgebra extensions and algebra coextensions of Galois type, *Comm. Algebra*, **27** (1999), 13471367.

[49] Brzeziński, T., Wisbauer, R. - Corings and comodules, London Math. Soc. Lect. Note Ser., **309** (2003), Cambridge University Press.
[50] Burde, D. - Derived length and nildecomposable Lie algebras, arXiv:1212.3113,
[51] Burde, D. and Ceballos, M - Abelian ideals of maximal dimension for solvable Lie algebras, *Journal of Lie Theory*, **22** (2012), 741–756.
[52] Byott, N. P. - Cleft extensions of Hopf algebras, *J. Algebra*, **157**(1993), 405–429.
[53] Byott, N. P. - Cleft extensions of Hopf algebras II, *Proc. London Math. Soc.* **67**(1993), 277–304.
[54] Caenepeel, S., Militaru G. and Zhu, S. - Frobenius and Separable Functors for Generalized Module Categories and Nonlinear Equations, *Lect. Notes Math.* **1787** Springer Verlag, Berlin, 2002.
[55] Calderon Martin, A, J. and Sanchez Delgado, J. M. - On split Leibniz algebras, *Linear Algebra Appl.*, **436** (2012), 1651–1663.
[56] Camacho, L.M., Cañetea, E.M., Gómez, J.R. and Redjepov, Sh.B. - Leibniz algebras of nilindex $n-3$ with characteristic sequence $(n-3,2,1)$, *Linear Algebra Appl.*, **438** (2013), 1832–1851.
[57] Canete, E.M and Khudoyberdiyev, A. Kh. - The classification of 4-dimensional Leibniz algebras, arXiv:1301.7665.
[58] Casas, J.M. and Corral, N. - On universal central extensions of Leibniz algebras, *Comm. Algebra*, **37** (2009) 2104-2120.
[59] Casas, J.M., Ladra, M., Omirov, B. A. and I.A. Karimjanov, I.A. - Classification of solvable Leibniz algebras with null-filiform nilradical, *Linear Multilinear Algebra*, **61** (2013), 758–774.
[60] Casas, J.M., Insua, M.A., Ladra, M. and Ladra S. - An algorithm for the classification of 3-dimensional complex Leibniz algebras, *Linear Algebra Appl.*, **436** (2012), 3747–3756.
[61] Ceballos, M. and Towers, D. A. - On abelian subalgebras and ideals of maximal dimension in supersolvable Lie algebras, *J. Pure Appl. Algebra*, **218** (2014), 497-503.
[62] Chekhov, L. and Mazzocco, M. - Poisson Algebras of Block-Upper-Triangular Bilinear Forms and Braid Group Action, *Comm. Math. Physics* **322** (2013), 49–71.
[63] Chen, Yongshan and Chen, Yuqun - Gröbner-Shirshov bases for metabelian Lie algebras, *J. Algebra*, **358** (2012), 143-161.
[64] Chevalley, C. and Eilenberg S. - Cohomology theory of Lie groups and Lie algebras, *Trans. Amer. Math. Soc.*, **63**(1948), 85–124.
[65] Conder, M. D. E. and Isaacs, I. M. - Derived subgroups of products of an abelian and a cyclic subgroup, *J. London Math. Soc.*, **69**(2004), 333-348.
[66] Covez, S. - The local integration of Leibniz algebras, *Ann. Inst. Fourier*, **63**(2013), 1-35.
[67] Cuvier, C. - Algèbres de Leibniz: définitions, propriétés, *Ann. Scient. Ec. Norm. Sup.*, **27** (1994), 1–45.
[68] Dascalescu, S., Nastasescu, C. and Raianu, S. - Hopf algebras. An introduction, Marcel Dekker, New York, 2000.
[69] Dangovski, R., Drensky, V. and Findik, Ş. - Weitzenböck derivations of free metabelian Lie algebras, *Linear Algebra Appl.* **439** (2013), 3279-3296.
[70] Daniyarova, E., Kazachkov I and Remeslennikov, V. - Algebraic geometry over free metabelian Lie algebra II: Finite field case, *J. Math. Sci.*, **135**(2006), 3311–3326.
[71] de Graaf, W.A. - Classification of Solvable Lie Algebras, *Experimental Mathematics*, **14** (2005), 15–25.
[72] de Graaf, W.A. - Classification of 6-dimensional nilpotent Lie algebras over fields of characteristic not 2, *J. Algebra*, **309**(2007), 640-653.
[73] del Barcoa, V. J. and Ovandob, G. P. - Free nilpotent Lie algebras admitting ad-invariant metrics, *J. Algebra*, **366** (2012), 205-216.
[74] Demir, I., Misra, K. C. and Stitzinger, E. - On some structures of Leibniz algebras, arXiv:1307.7672.
[75] Doi, Y. and Takeuchi, M. - Cleft comodule algebras for a bialgebra, *Comm. Algebra*, **14** (1986), 801 – 818.
[76] Douglas, J. - On finite groups with two independent generators. I,II, III, IV. *Proc. Nat. Acad. Sci. U. S. A.*, **37** (1951), 604-610, 677-691, 749-760, 808-813.
[77] Drensky, V. and Piacentini Cattaneo G. M. - Varieties of metabelian Leibniz algebras, *J. Algebra Appl.* **01**, 31 (2002).

[78] Drinfeld, V. - Quantum groups, *Proc. Intern. Congr. Math., Berkeley*, Vol. I (1987), 798 – 820.
[79] Erdmann, K. and Wildon, M.J. - Introduction to Lie algebras, Springer, 2006.
[80] Farnsteiner, R. - On the cohomology of associative algebras and Lie algebras, *Proc. AMS*, **99**(1987), 415-420.
[81] Fialowski, A., Khudoyberdiyev, A. Kh. and Omirov, B. A. - A characterization of nilpotent Leibniz algebras, *Algebr. Represent. Theor.*, **16** (2013), 1489-1505.
[82] Fialowski, M. and Mandal, A. - Leibniz algebra deformations of a Lie algebra, *J. Math. Physics*, **49**(2008), 093512, 10 pages.
[83] Fialowski, A., Penkava, M. - The moduli space of 3-dimensional associative algebras, *Comm. Algebra*, **37** (2009), 3666–3685.
[84] Fialowski, A., Penkava, M., Phillipson, M. - Deformations of complex 3-dimensional associative algebras, *J. Gen. Lie Theory Appl.*, **5** (2011), Art. ID G110102, 22 pp.
[85] Figueroa-O´Farrill, J. M., Stanciu, S. - On the structure of symmetric self-dual Lie algebras, *J. Math. Phys.*, **37** (1996), 4121-4134.
[86] Fisher, D.J., Gray, R. J. and Hydon, P. E. - Automorphisms of real Lie algebras of dimension five or less, *Journal of Physics A: Mathematical and General* **46** (2013), 225204.
[87] Galitski, L. Yu. and Timashev, D. A. - On classification of metabelian Lie algebras, *J. Lie Theory*, **9** (1999), 125–156.
[88] Garcia, G. A., Vay, C. - Hopf algebras of dimension 16, *Algebr. Represent. Theory*, **13** (2010), 383–405.
[89] Gauger, M. - On the classification of metabelian Lie algebras, *Trans. Amer. Math. Soc.*, **179**(1973), 293–329.
[90] Gavarini, F. - The crystal duality principle: from Hopf algebras to geometrical symmetries, *J. Algebra* **285** (2005), 399–437.
[91] Gorbatsevich, V. V. - On the level of some solvable Lie algebras, *Siberian Math. J.* **39** (1998), 872-883.
[92] Gorbatsevich, V. V. - On some basic properties of Leibniz algebras, arXiv:1302.3345.
[93] Grabowski, J. - Abstract Jacobi and Poisson structures, *J. Geom. Phys.* **9** (1992), 45-73.
[94] Grabowski, J. Marmo, G. and Perelomov, A. M. - Poisson structures: towards a classification, *Modern Phys. Lett. A* **8**(1993), 1719–1733.
[95] Grabowski, J. and Marmo, G. - The graded Jacobi algebras and (co)homology, *J. Phys. A* **36** (2003), 161–181.
[96] Grabowski, J. - Brackets, *Int. J. Geom. Methods Mod. Phys.*, 10(8):1360001, 45, 2013.
[97] Hofmann, K. H. - Lie algebras with subalgebras of codimension one, *Illinois J. Math.*, **9** (1965), 636–643.
[98] Hofstetter, I - Extensions of Hopf algebras and thier cohomological description, *J. Algebra*, **164** (1994), 264–298.
[99] Hu, N., Pei, Y. and Liu, D. - A cohomological characterization of Leibniz central extensions of Lie algebras, *Proc. AMS*, **136**(2008), 437-447
[100] Humphreys, J.E. - Introducution to Lie algebras and Representation theory, Spinger, 1972.
[101] Iovanov, M.C. - Generalized Frobenius algebras and Hopf algebras, *Canad. J. Math.* **66** (2014), 205-240.
[102] Itô, N. - Über das produkt von zwei abelschen gruppen, *Math. Z.*, **62** (1955) 400-401.
[103] Jacobson, N. - Lie Algebras, Interscience Publ., New YorkLondon, 1962.
[104] Jiang, C.P., Meng, D.J. and Zhang S.Q. - Some complete Lie algebras, *J. Algebra*, **186**(1996), 807-817.
[105] Kadison, L. - New Examples of Frobenius Extensions, Univ. Lect. Series **14** (1999), Amer. Math. Soc., Providence.
[106] Kassel, C. - Quantum groups, Graduate Texts in Mathematics 155. Springer-Verlag, New York, 1995.
[107] Kath, I. and Olbrich, M. - Metric Lie algebras with maximal isotropic centre, *Math. Z.* **246** (2004) pp 23–53.

[108] Khudoyberdiyev., A.Kh., Omirov, B.A. - The classification of algebras of level one, *Linear Algebra Appl.* **439** (2013), 3460–3463.
[109] Kinyon, M. K. and Weinstein, A. - Leibniz algebras, Courant algebroids, and multiplications on reductive homogeneous spaces, *Amer. J. Math.*, **123**(2001), 525–550.
[110] Kirillov, A. - Local Lie Algebras, Russian Math. Surveys **31** (1976), 55-75.
[111] Kock, J. - Frobenius algebras and 2D topological quantum field theories, London Math. Soc. **59**(2003), Cambridge University Press.
[112] Kurdiani, R. and Pirashvilli, T. - A Leibniz algebra structure on the second tensor power, *J. of Lie Theory*, **12**(2002), 583–596.
[113] Lambre, T., Zhou, G. and Zimmermann, A. - The Hochschild cohomology ring of a Frobenius algebra with semisimple Nakayama automorphism is a Batalin-Vilkovisky algebra, arXiv:1405.5325.
[114] Laurent-Gengoux, C., Pichereau, A. and Vanhaecke, P. - Poisson Structures, Vol. 347, 2013, Springer.
[115] Lecomte, P. - Sur la suite exacte canonique asociée à un fibrè principal, *Bul. Soc. Math. France*, **13**(1985), 259–271.
[116] Lichnerowicz, A. - Le variétés de Jacobi et leurs algèbres de Lie associées, *J. Math. Pures Appl.* **57(4)** (1978), 453–488.
[117] Lie, S. - Theorie der Transformationsgruppen, Vol. 1-3, Leipzig, 1888, 1890, 1893.
[118] Loday, J.-L. - Une version non commutative des algèbres de Lie: les algèbres de Leibniz, *L'Enseignement Math.*, **39** (1993), 269–293.
[119] Loday, J.-L. and Pirashvili, T. - Universal enveloping algebras of Leibniz algebras and (co)homology, *Math. Ann.*, **296**(1993), 139–158.
[120] Lu, J., Wang, X. and Zhuang, G. - Universal enveloping algebras of Poisson Hopf algebras, *J. Algebra* **426** (2015), 92-136.
[121] Lu, J.H. and Weinstein, A. - Poisson Lie groups, dressing transformations and Bruhat decompositions, *J. Differential Geom.*, **31**(1990), 501–526.
[122] Majid, S. - Physics for algebraists: non-commutative and non-cocommutative Hopf algebras by a bicrossproduct construction, *J. Algebra*, **130** (1990), 17–64.
[123] Majid, S. - Matched Pairs of Lie Groups and Hopf Algebra Bicrossproducts, *Nuclear Physics B, Proc. Supl.* **6** (1989), 422–424.
[124] Majid, S. - Foundations of quantum groups theory, Cambridge University Press, 1995.
[125] Majid, S. and Soibelman, Ya. S. - Bicrossproduct structure of the quantum Weyl group, *J. Algebra* **163** (1994), 68 – 87.
[126] Marle, C. M. - On Jacobi Manifolds and Jacobi boundles, In *Symplectic Geometry, Groupoids, and Integrable Systems*, Mathematical Sciences Research Institute Publications **20** (1991), 227–246, Springer.
[127] Mason, G. and Yamskulna, G. - Leibniz algebras and Lie algebras, arXiv:1201.5071.
[128] Masuoka, A. - Extensions of Hopf algebras, Trabajos de Matematica 41/99 Fa.M.A.F. (1999).
[129] Masuoka, A. - Abelian and non-abelian second cohomologies of quantized enveloping algebras, *J. Algebra*, **320** (2008), 1–47.
[130] Mazzola, G. - The algebraic and geometric classification of associative algebras of dimension five, *Manuscripta Math.* **27** (1979), 81-101.
[131] Medina, A., Revoy, Ph. - Algébres de Lie et produit scalaire invariant, *Ann. Sci. Ècole Norm. Sup.*, **18** (1985), 553-561.
[132] Michor, P. W. - Knit products of graded Lie algebras and groups, Proceedings of the Winter School on Geometry and Physics (Srn, 1989). *Rend. Circ. Mat. Palermo* (2) Suppl. No. **22** (1990), 171–175.
[133] Militaru, G. - The global extension problem, co-flag and metabelian Leibniz algebras, to appear in *Linear Multilinear Algebra*, DOI:10.1080/03081087.2014.891587.
[134] Molnar, R.K. - Semi-direct products of Hopf algebras, *J. Algebra* **47** (1977), 29 – 51.
[135] Montgomery, S. - Hopf algebras and their actions on rings, vol. 82 of CBMS Regional Conference Series in Mathematics, AMS, Providence, Rhode Island (1993).
[136] Xiang Ni and Chengming Bai - Poisson bialgebras, *J. Math. Phys.*, **54**, 023515 (2013).

[137] Omirov, B.A., Rakhimov, I. S. and Turdibaev, R. M. - On description of Leibniz algebras corresponding to $\mathfrak{sl}_2$, *Algebr. Represent. Theor.*, **16** (2013), 1507-1519.
[138] Ovando, G. - Four Dimensional Symplectic Lie Algebras, Beiträge zur Algebra und Geometrie, **47**(2006), 419–434.
[139] Panaite, F. - Equivalent crossed products and cross product bialgebras, *Comm. Algebra*, DOI:10.1080/00927872.2012.751538, arXiv:1203.0037.
[140] Pelc, O. - A New Family of Solvable Self-Dual Lie Algebras, *J.Math.Phys.*, **38**(1997), 3832–3840.
[141] Petravchuk, A. P. - On the sum of an almost abelian Lie algebra and a Lie algebra finite dimensional over its center, *Ukrain. Math. J.*, **51** (1999), 707-715.
[142] Pierce, R.S. - Associative Algebras, Springer-Verlag, 1982.
[143] Popovych, R.O., Boyko, V.M., Nesterenko, M.O. and Lutfullin, M. W. – Realizations of real low-dimensional Lie algebras, *Journal of Physics A: Mathematical and General* **36** (2003), 7337.
[144] Poroshenko, E. N. and Timoshenko, E. I. - Universal equivalence of partially commutative metabelian Lie algebras, *J. Algebra*, **384** (2013), 143-168.
[145] Radford, D. E. - Hopf algebras, World Scientific Publishing, Series on knots and everything, vol. **49** (2012).
[146] Rakhimov, I.S. and Hassan, M.A. - On isomorphism criteria for Leibniz central extensions of a linear deformation of μ_n, *Internat. J. Algebra Comput.*, **21** (2011) 715-729.
[147] Rakhimov, I.S. and Husain, S.K. - Classification of a subclass of nilpotent Leibniz algebras, *Linear and Multilinear Algebra*, **59** (2011) 339-354.
[148] Rakhimov, I.S., Rikhsiboev, I.M., Basri, W. - Complete lists of low dimensional complex associative algebras, arXiv:0910.0932v2.
[149] Rotman, J.J. - An introduction to the theory of groups. Fourth edition. Graduate Texts in Mathematics 148, Springer-Verlag, New York, 1995.
[150] Rovi, A. - Hopf algebroids associated to Jacobi algebras, *Int. J. Geom. Methods Mod. Phys.* **11**, 1450092 (2014).
[151] Schauenburg, P. - The structure of Hopf algebras with a weak projection, *Algebr. Represent. Theory* **3** (1999), 187 –211.
[152] Schneider, H.-J. - Some remarks on exact sequences of quantum groups, *Comm. Algebra* **21**(1993), 3337–3358.
[153] Singer, W. - Extension theory for connected Hopf algebras, *J. Algebra*, **21**(1972), 1-16.
[154] Study, E. - Über Systeme complexer Zahlen und ihre Anwendungen in der Theorie der Transformationsgruppen, *Monatshefte für Mathematik* **1** (1890), 283-354.
[155] Su, Y. and Zhub, L. - Derivation algebras of centerless perfect Lie algebras are complete, *J. Algebra*, **285** (2005), 508–515.
[156] Sweedler, M.E. - Cohomology of algebras over Hopf algebras, *Trans. AMS*, **133** (1968), 205–239.
[157] Sweedler, M.E. - Hopf Algebras, Benjamin New York, 1969
[158] Szechtman, F. - Equivalence and normal forms of bilinear forms, *Linear Algebra Appl.*, **443** (2014), 245-259.
[159] Takeuchi, M. - Matched pairs of groups and bismash products of Hopf algebras, *Comm. Algebra* **9**(1981), 841–882.
[160] Turkowski, P. - Solvable Lie algebras of dimension six, *J. Math. Phys.*, **31**(1990), 1344-1350.
[161] Vaes, S. and Vainerman, L. Extensions of locally compact quantum groups and the bicrossed product construction, *Adv. Math.* **175** (2003), 1101.
[162] Zhu, C., Van Oystaeyen, F. and Zhang Y. - On the (co) homology of Frobenius Poisson algebras, *Journal of K-theory: K-theory and its Applications to Algebra, Geometry, and Topology* **14** (2014), 371–386.
[163] Zusmanovich, P. - Central extensions of current algebras, *Trans. AMS*, **334**(1992), 143–152.